Name: ______________________

Period: ______________________

Rotational Motion

Angular Quantities

Radius doesn't matter. Must use radians.

θ (in rad)

Angular Displacement (*Theta*) – how much a circle turns or how much of an angle an object travels.

Example: A circle turns two revolutions. Find θ.
Solution: *Since 2π = 360° = 1 revolution.*

$$\theta = \frac{2\ \text{rev}}{1}\left(\frac{2\pi}{1\ \text{rev}}\right) = 4\pi$$

ω (in rad/s)
ω = Δθ/t

Angular Velocity (*Omega*) – how fast a circle turns or how fast an object moves through an angle

Example: A circle turns two revolutions in 4 secs. Find ω.
Solution: *2 rev = 4π rad = 12.6 rad*

$$\omega = \frac{\theta}{t} = \frac{12.6\text{ rad}}{4\text{ sec}} = 3.14\text{ rad/sec}$$

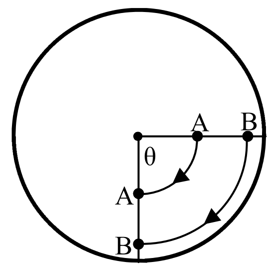
α (rad/s²)
α = Δω/t

Angular Acceleration (*Alpha*) – how fast a circle speeds up or how fast an object accelerates around a circle.

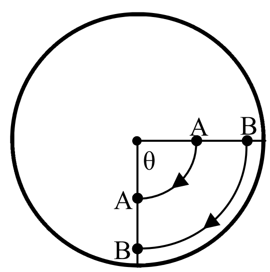
Example: A circle starts at rest. After 2 sec, it is spinning at 12 rad/sec. Find α.
Solution:

$$\alpha = \frac{\Delta\omega}{t} = \frac{\omega_f - \omega_i}{t} = \frac{12\text{ rad} - 0\text{ rad}}{2\text{ sec}} = 6\text{ rad/sec}^2$$

Point A and B have the same angular displacement: $\theta_A = \theta_B$. Yet, since $r_A < r_B$, point A travels a smaller arc length: $s_A < s_B$.



Point A and B have the same angular velocity: $\omega_A = \omega_B$ (they will take the same amount of time to turn the circle), but $v_{tA} < v_{tB}$.



A and B start at rest (ωi = 0), and after a time (t) the circle is turning with an angular speed (ω_f) then they experienced the same angular acceleration: $\alpha_A = \alpha_B$, but B experiences a greater tangential acceleration: $a_{tB} > a_{tA}$.

Tangential Quantities

Radius does matter.

s (in m)
s = rθ

Arc Length – how much of a circumference an object travels.

Example: A circle turns two revolutions. Find s for an object .3 m from the center of the circle.
Solution: *θ = 2 rev = 4π rad = 12.6 rad*

$$s = r\theta = .3(12.6) = 3.78\text{m}$$

v_t (in m/s)
$v_t = r\omega$

Tangential Velocity – velocity at a radius. How fast something is moving straight ahead (like a car's speedometer while going around a track).

Example: A car is going 0.7 rad/sec around an 83 m radius track. Find v_t.
Solution:

$$v_t = r\omega = (83)(0.7) = 58.1\text{ m/s}$$

a_t (in m/s²)
$a_t = r\alpha$

Tangential Acceleration – acceleration at a radius. How fast something is speeding up in straight ahead. A runner in the outside lane has to have more tangential acceleration to stay along a runner in the inside lane.

Example: Using the example at the left, find the tangential acceleration of a bug 0.35 m from the center of the circle.
Solution:

$$a_t = r\alpha = (0.35)(6) = 2.1\text{ m/s}^2$$

Kinematic Equations

The rotational kinematic equations are exactly the same as their linear counterparts, using the following substitutions:

x *becomes* θ
v *becomes* ω
a *becomes* α

Linear

$$\Delta x = \frac{1}{2}(v_i + v_f)\Delta t$$

$$v_f = v_i + a\Delta t$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$\Delta x = v_i(\Delta t) + \frac{1}{2}a(\Delta t)^2$$

$$\Delta x = v_f(\Delta t) - \frac{1}{2}a(\Delta t)^2$$

Rotational

$$\Delta\theta = \frac{1}{2}(\omega_i + \omega_f)\Delta t$$

$$\omega_f = \omega_i + \alpha\Delta t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \omega_i(\Delta t) + \frac{1}{2}\alpha(\Delta t)^2$$

$$\Delta\theta = \omega_f(\Delta t) - \frac{1}{2}\alpha(\Delta t)^2$$

Name: ____________________

Period: ____________________

1. Tangential Acceleration	A. How far an object moves along the edge of a circle.
2. Angular Displacement	B. Rate at which speed changes that is radius dependent.
3. Tangential Velocity	C. How fast an object moves at a fixed radius.
4. Arc Length	D. Rate at which a circle turns.
5. Angular Acceleration	E. Rate at which a circle changes speed.
6. Angular Velocity	F. How much of a circle an object moves.

7. θ, ω, α, s, v_t, or a_t

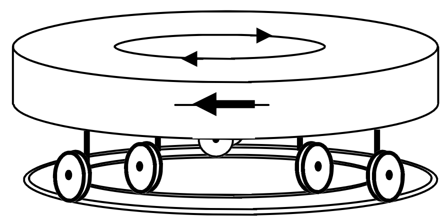
A. ____ In m/s
B. ____ In rad/sec
C. ____ In rad
D. ____ In m/s^2
E. ____ In rad/sec^2
F. ____ In m

G. Which ones are radius dependent?
H. Which ones are radius independent?

8. A) Convert 3 revolutions to radians.

B) Convert 20 rpm (rev per min) to rad/sec.

9. *Use the graphic of the rotating platform at the right to answer the following . Possible Answers: I (Inside wheels); O (outside wheels); S (same or both).*



A platform turning clockwise, when viewed from above.

A) ____ Greatest radius?
B) ____ Fastest tangential speed?
C) ____ If it comes to rest, which have the fastest α?
D) ____ Travels the least arc length?
E) ____ Which has the smallest radius?
F) ____ Which have the fastest angular speed?
G) ____ As it starts rotating, which has the slowest a_t?
H) ____ Has the slowest tangential speed?
I) ____ Travels the greatest angular displacement?
J) ____ Has the slowest angular velocity?
K) ____ As it slows, which has the greatest a_t?
L) ____ Travels the longest arc length?

The following two columns are designed to help you see the correlations between linear and rotational quantities and equations. Do the problems in number order to see the relationships.

10. A car travels 240 meters in 12 seconds. Find the velocity of the car.

11. A wheel rotates 2 revolutions in 3 seconds. Find the angular velocity of the wheel.

12. A car going 300 m/s slows to 100 m/s in 10 seconds. Find the acceleration of the car.

13. A wheel spinning 8 rad/sec slows to 2 rad/sec in 3 seconds. Find the angular acceleration of the wheel.

14. A car going 20 m/s stops in 80 meters. How long did it take to stop?

15. A wheel turning 24 rad/sec stops in 6 radians. How long did it take to stop?

16. A box sliding down a hill going 3 m/s accelerates at 2 m/s^2. How fast is going after 4 seconds?

17. A wheel turning 2 rad/sec accelerates at 3 rad/sec^2. How fast is it spinning after 5 seconds?

1. Tangential Acceleration B
2. Angular Displacement F
3. Tangential Velocity C
4. Arc Length A
5. Angular Acceleration E
6. Angular Velocity D

A. How far an object moves along the edge of a circle.

B. Rate at which speed changes that is radius dependent.

C. How fast an object moves at a fixed radius.

D. Rate at which a circle turns.

E. Rate at which a circle changes speed.

F. How much of a circle an object moves.

7. θ, ω, α, s, v_t, or a_t

A. v_t In m/s
B. ω In rad/sec
C. θ In rad
D. a_t In m/s^2
E. α In rad/sec^2
F. s In m

G. Which ones are radius dependent? s, v_t, a_t
H. Which ones are radius independent? θ, ω, α

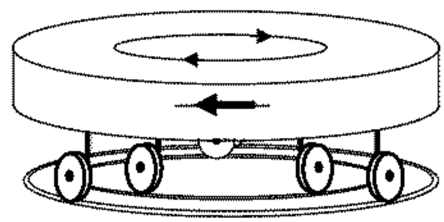
8. A) Convert 3 revolutions to radians.

$\left(\frac{3\ rev}{1}\right)\left(\frac{2\pi\ rad}{1\ rev}\right) = 6\pi\ rad = 18.84\ rad$

B) Convert 20 rpm (rev per min) to rad/sec.

$\frac{20\ rev}{1\ min}\left(\frac{1\ min}{60\ sec}\right)\left(\frac{2\pi\ rad}{1\ rev}\right) = .66\pi\ rad/sec$
$= 2.07\ rad/sec$

9. *Use the graphic of the rotating platform at the right to answer the following . Possible Answers: I (Inside wheels); O (outside wheels); S (same or both).*



A platform turning clockwise, when viewed from above.

A) O Greatest radius?
B) O Fastest tangential speed?
C) S If it comes to rest, which have the fastest α?
D) I Travels the least arc length?
E) I Which has the smallest radius?
F) S Which have the fastest angular speed?
G) I As it starts rotating, which has the slowest a_t?
H) I Has the slowest tangential speed?
I) S Travels the greatest angular displacement?
J) S Has the slowest angular velocity?
K) O As it slows, which has the greatest a_t?
L) O Travels the longest arc length?

The following two columns are designed to help you see the correlations between linear and rotational quantities and equations. Do the problems in number order to see the relationships.

10. A car travels 240 meters in 12 seconds. Find the velocity of the car.

$V = \frac{\Delta D}{T} = \frac{240\ m}{12\ sec} = 20\ m/s$

11. A wheel rotates 2 revolutions in 3 seconds. Find the angular velocity of the wheel.

$\frac{2\ rev}{1}\left(\frac{2\pi\ rad}{1\ rev}\right) = 4\pi\ rad \quad \omega = \frac{\Delta\theta}{t} = \frac{4\pi}{3} = 4.2\ rad/sec$

12. A car going 300 m/s slows to 100 m/s in 10 seconds. Find the acceleration of the car.

$a = \frac{\Delta v}{t} = \frac{v_f - v_i}{t} = \frac{100-300}{10} = \frac{-200}{10} = -20\ m/s^2$

13. A wheel spinning 8 rad/sec slows to 2 rad/sec in 3 seconds. Find the angular acceleration of the wheel.

$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{2-8}{3} = \frac{-6}{3} = -2\ rad/sec^2$

14. A car going 20 m/s stops in 80 meters. How long did it take to stop?

$\Delta x = \frac{1}{2}(v_i + v_f)t$
$80 = \frac{1}{2}(20+0)t$
$80 = 10t$
$8\ sec = t$

15. A wheel turning 24 rad/sec stops in 6 radians. How long did it take to stop?

$\Delta\theta = \frac{1}{2}(\omega_f + \omega_i)t$
$6 = \frac{1}{2}(0+24)t$
$12 = 24t$
$0.5\ sec = t$

16. A box sliding down a hill going 3 m/s accelerates at 2 m/s^2. How fast is going after 4 seconds?

$V_f = V_i + at$
$V_f = 3 + 2(4)$
$V_f = 3 + 8$
$= 11\ m/s$

17. A wheel turning 2 rad/sec accelerates at 3 rad/sec^2. How fast is it spinning after 5 seconds?

$\omega_f = \omega_i + \alpha t$
$\omega_f = 2 + 3(5)$
$\omega_f = 2 + 15 = 17\ rad/sec$