Rotational Motion

- Moment of Inertia → depends on the mass distribution and the axis of rotation of an object. The ones here are the most commonly used, and the first one is the common shape for a pulley.

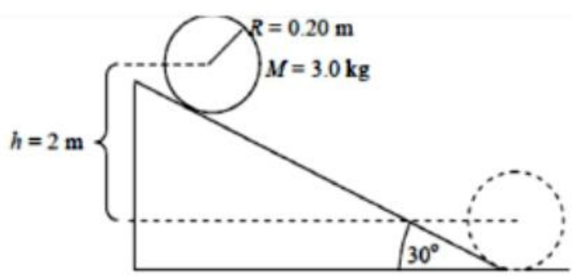
Cylinder:	$I = \frac{1}{2}MR^2$
Cylindrical hoop:	$I = MR^2$
Sphere:	$I = \frac{2}{5}MR^2$

- Kinetic Energy → Objects that roll will have some rotational kinetic energy. Objects that slide WILL NOT have rotational kinetic energy, and thus should be moving faster than objects that roll.

What is the kinetic energy of the sphere at the bottom of the incline? The sphere is translating and rotating at the same time. The total kinetic energy is the addition of the translational and rotational kinetic energies.

$$K_{total} = K_{translation} + K_{rotation}$$

$$K_{total} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$



It is also equal to the potential energy that converted in kinetic energy. Given the quantities in the problem, this is easiest to solve

$$K_{total} = mgh \qquad K_{total} = (3.0)(9.8)(2.0) = \boxed{58.8J}.$$

How fast is the sphere going at the bottom of the incline?

Now the first kinetic energy equation has relevance, $K_{total} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$. Combine this with the moment of inertia equation of a sphere $\frac{2}{5}MR^2$, and the equation $\omega = \frac{v}{R}$ which converts angular values into linear values.

$$K_{total} = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2 \qquad K_{total} = \frac{1}{2}Mv^2 + \frac{2}{10}Mv^2 \qquad K_{total} = \frac{7}{10}Mv^2$$

Rearrange for velocity, plug in values, and solve.

$$v = \sqrt{\frac{10}{7}\frac{K_{total}}{M}} \qquad v = \sqrt{\frac{10}{7}\left(\frac{58.8}{3.0}\right)} = \boxed{5.29\,m/s}$$

Angular momentum: Depends on mass (like regular momentum) and it also depends on mass distribution. As an ice skater brings their arms closer to the body they begin to spin faster, since the mass has a shorter distance to travel.

Angular momentum is conserved. The radius gets smaller, but angular velocity increases (vice versa as the skater moves arms outward). A galaxy, solar system, star, or planet forms from a larger cloud of dust. As the cloud is pulled together by gravity its radius shrinks. So the angular velocity must increase. These objects all begin to spin faster. That is why we have day and night.

Torque: In rotation problems we look at the sum of torque (not the sum of force). But it is exactly the same methodology.

$\boxed{\tau = rF\sin\theta}$ Strongest when the force is **perpendicular** to the lever arm (since sin 90° equals one).

Balanced Torque: The sum of torque is zero. No rotation.

Unbalance Torque: Adding all the clockwise and counterclockwise torque does not sum to zero. So there is excess torque in either the clockwise or counterclockwise direction. This will cause the object to rotate.

1. **As always, ask what the object is doing.** Is it rotating or is it standing still?
2. **Set the direction of motion as positive.** The convention when in doubt is that counterclockwise is positive. This corresponds to projectile motion where angles measured from the horizon counterclockwise were positive. But, just like in forces if you know the direction of motion call it positive. It will either rotate clockwise or counterclockwise. If you pick the wrong direction your final answer will be negative. But, the answer will be correct nonetheless. If it is not moving pick one direction to be positive, it really doesn't matter. But the other must be negative, so the opposing torques cancel.
3. **Identify the sum of torque equation.**

$\sum\tau = \sum\tau_{cw} - \sum\tau_{ccw}$ **or** $\sum\tau = \sum\tau_{ccw} - \sum\tau_{cw}$

4. **Substitute the relevant force equations and solve** (examples assume clockwise was positive direction)

Rotating: $\sum\tau = \sum(rF\sin\theta)_{cw} - \sum(rF\sin\theta)_{ccw}$

Not Rotating: $0 = \sum(rF\sin\theta)_{cw} - \sum(rF\sin\theta)_{ccw}$ $\sum(rF\sin\theta)_{cw} = \sum(rF\sin\theta)_{ccw}$

Example 10-1: Torque and a Seesaw

Three masses are positioned on a seesaw as shown in Fig 10.1. $m_A = 4.0$ kg, $m_B = 2.0$ kg, and $m_3 = 3.0$ kg. Distances are shown in the diagram.

How far from the fulcrum must m_B be positioned in order for the system to balance? Keep in mind that measurements are made from the center of mass. It is as though all the mass is mathematically located at a point at the center of the object. It is not rotating, so the clockwise torques must equal the counterclockwise torques.

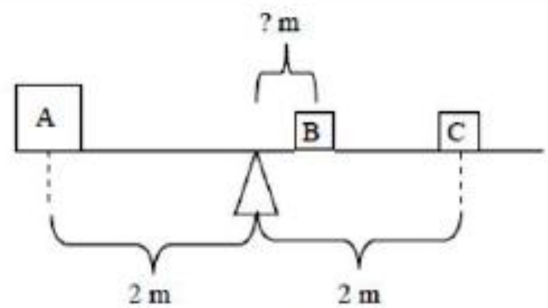


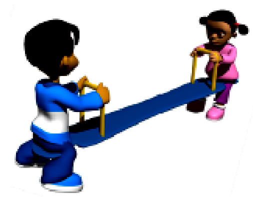
Fig 10.1

$$\sum\tau_{cw} = \sum\tau_{ccw}$$
$$\tau_A = \tau_B + \tau_C$$
$$r_A \cdot F_A = r_B \cdot F_B + r_C \cdot F_C$$
$$r_A \cdot m_A \cancel{g} = r_B \cdot m_B \cancel{g} + r_C \cdot m_C \cancel{g}$$
$$(2)(4) = r_B(2) + (2)(3) \qquad r_B = \boxed{1.0m}$$

Rotation: Since every point on a rotating object experiences a different tangential velocity displacement, velocity, and acceleration cannot be expressed in terms of meters. A particle on the outside edge of a rotating object covers a greater distance in the same time interval than a particle closer to the center. The only quantity that both points share in any given time interval is the angle through which they move, as shown to the right. In rotation we have to work in radians instead of degrees. This means that for every variable in linear (***translational***) motion there is a corresponding variable for rotation. And every equation in linear motion has a rotational counterpart. Displacement x is replaced by ***radians*** θ (radians). Velocity v is replaced by ***angular velocity*** ω (radians per second). Acceleration a is replaced by ***angular acceleration*** α (radians per second squared) The following three equations form a bridge between linear motion and rotation and should be memorized. $\boxed{x = r\theta}$ $\boxed{v = r\omega}$ $\boxed{a = r\alpha}$. The chart below, and on the following pages, compares rotation to linear motion. There is an analogous quantity and an analogous equation for rotation that parallels those learned in linear translational motion. Keep the three equations listed above in mind and become familiar with the new quantities.

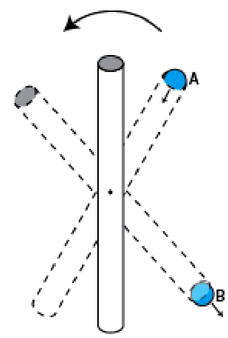
	Angular	Linear
Position	$\theta = \frac{arc\ length}{r}$	x
Displacement	$\Delta\theta = \theta - \theta_0$	$\Delta x = x - x_0$
Average Speed	$\bar{\omega} = \frac{\Delta\theta}{\Delta t}$	$\bar{v} = \frac{\Delta x}{\Delta t}$ $\bar{v} = \frac{v_0 + v}{2}$
Instantaneous Speed	$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$ $\omega = \frac{d\theta}{dt}$	$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$ $v = \frac{dx}{dt}$ **Slope of displacement - time graph**
Average Acceleration	$\bar{\alpha} = \frac{\Delta\omega}{\Delta t}$	$\bar{a} = \frac{\Delta v}{\Delta t}$
Instantaneous Acceleration	**Tangential Acceleration** $\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$ $\alpha = \frac{d\omega}{dt}$	$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$ $a = \frac{dv}{dt}$ **Slope of velocity - time graph**
Kinematic Equations	$\omega = \omega_0 + \alpha t$ $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$	$v = v_0 + at$ $x = x_0 + v_0 t + \frac{1}{2}at^2$ $v^2 = v_0^2 + 2a(x - x_0)$
Tangential Speed	$v = r\omega$ $\omega = \frac{2\pi}{T}$	$v = \frac{2\pi r}{T}$
Centripetal Acceleration	**Radial Acceleration** $a_c = \frac{v^2}{r} = \omega^2 r$ Radial Acceleration is the acceleration directed along a radial (spoke) line. It is directed toward the center.	$a_c = \frac{v^2}{r}$
Inertia	**Moment of Inertia:** Depends on mass and distribution and thus varies for each object $I = \int r^2 dm = \sum mr^2$ Since these vary from object to object they are usually given. The three shown here are commonly used. The first one is the common shape for pulley, which are the most used. Cylinder: $I = \frac{1}{2}MR^2$ Cylindrical hoop: $I = MR^2$ Sphere: $I = \frac{2}{5}MR^2$	m
Force and Torque	**Torque:** Unbalance torques cause rotation. $\tau = \mathbf{r} \times \mathbf{F}$ $\sum \tau = \tau_{net} = I\alpha$	**Force:** Unbalanced forces cause translation. $\mathbf{F}$ $\sum \mathbf{F} = \mathbf{F}_{net} = m\mathbf{a}$
Kinetic Energy	$K = \frac{1}{2}I\omega^2$	$K = \frac{1}{2}mv^2$

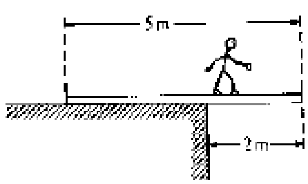
1. A 50-kg boy and a 40-kg girl sit on opposite ends of a 3-meter see-saw. How far from the girl should the fulcrum be placed in order for the boy and girl to balance on opposite ends of the see-saw?



2. A uniform hollow tube of length L rotates vertically about its center of mass as shown. A ball is dropped into the tube at position A, and exits a short time later at position B. From the perspective of a stationary observer watching the tube rotate, the distance the ball travels is

(A) less than L
(B) greater than L
(C) equal to L





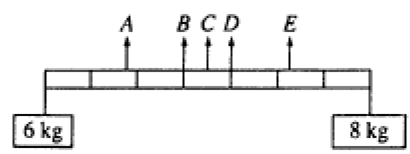
37. A 5-meter uniform plank of mass 100 kilograms rests on the top of a building with 2 meters extended over the edge as shown above. How far can a 50-kilogram person venture past the edge of the building on the plank before the plank just begins to tip?

A) $\frac{1}{2}m$ B) 1 m

C) $\frac{2}{3}m$ D) 2 m

E) It is impossible to make the plank tip since the person would have to he more than 2 meters from the edge of the building.

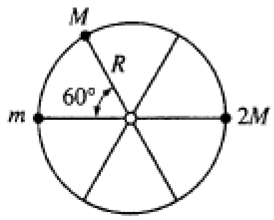
Problem: Rotational Equilibrium (1993)



36. Two objects, of masses 6 and 8 kilograms, are hung from the ends of a stick that is 70 centimeters long and has marks every 10 centimeters, as shown above. If the mass of the stick is negligible, at which of the points indicated should a cord be attached if the stick is to remain horizontal when suspended from the cord?

(A) A (B) B (C) C (D) D (E) E

Problem: Rotational Equilibrium (1998)



38. A wheel of radius R and negligible mass is mounted on a horizontal frictionless axle so that the wheel is in a vertical plane. Three small objects having masses m, M, and $2M$, respectively, are mounted on the rim of the wheel, as shown above. If the system is in static equilibrium, what is the value of m in terms of M?

(A) $\frac{M}{2}$

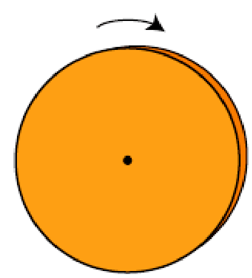
(B) M

(C) $\frac{3M}{2}$

(D) $2M$

(E) $\frac{5m}{2}$

4. A disc rotates clockwise about its axis as shown in the diagram. The direction of the angular momentum vector is:



(A) out of the plane of the page
(B) into the plane of the page
(C) toward the top of the page
(D) toward the bottom of the page

6. Jean stands at the exact center of a large spinning frictionless uniform disk of mass M and radius R with moment of inertia $I=\frac{1}{2}MR^2$. As she walks from the center to the edge of the disk, the angular speed of the disk is quartered. Which of the following statements is true?

(A) Jean's mass is less than the mass of the disk.
(B) Jean's mass is equal to the mass of the disk.
(C) Jean's mass is between the mass of the disk and twice the mass of the disk.
(D) Jean's mass is more than twice the mass of the disk.

7. A spinning plate in a microwave with moment of inertia I rotates about its center of mass at a constant angular speed ω. When the microwave ends its cook cycle, the plate comes to rest in time Δt due to a constant frictional force F applied a distance r from the axis of rotation. What is the magnitude of the frictional force F?

(A) $F = \dfrac{I\omega}{r\Delta t}$

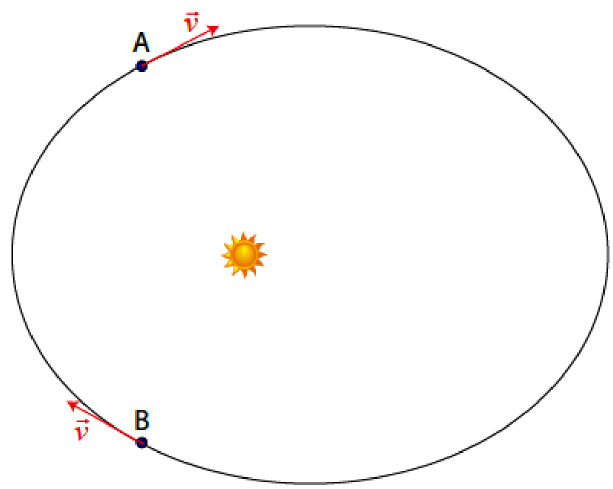
(B) $F = \dfrac{Ir}{\omega\Delta t}$

(C) $F = \dfrac{\omega r^2}{I\Delta t}$

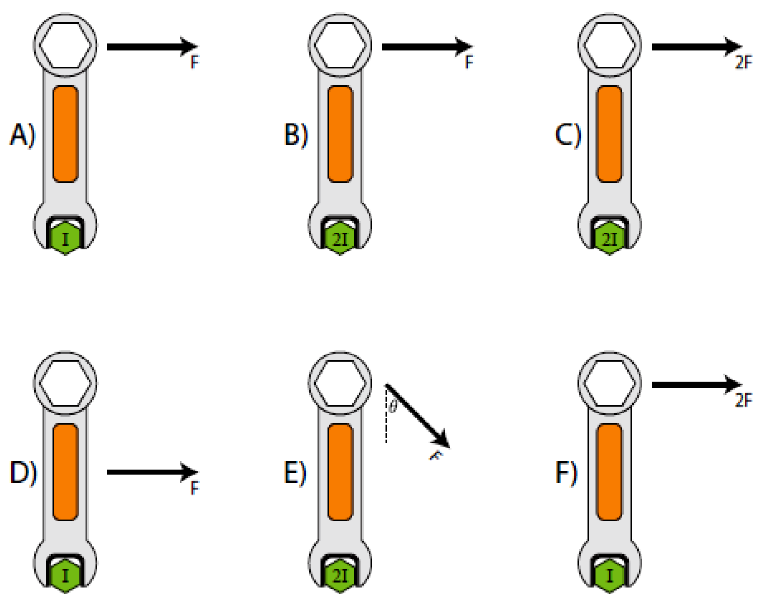
(D) $F = \dfrac{Ir^2}{\omega\Delta t}$

8. A planet orbits a sun in an elliptical orbit as shown. Which principles of physics most clearly and directly explain why the speed of the planet is the same at positions A and B? Select two answers.

(A) Conservation of Energy
(B) Conservation of Angular Velocity
(C) Conservation of Angular Momentum
(D) Conservation of Charge



9. A given force is applied to a wrench to turn a bolt of specific rotational inertia I which rotates freely about its center as shown in the following diagrams. Which of the following correctly ranks the resulting angular acceleration of the bolt?



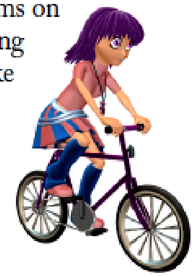
(A) $\alpha_F > \alpha_A = \alpha_C > \alpha_B = \alpha_D > \alpha_E$

(B) $\alpha_C = \alpha_F > \alpha_A = \alpha_B > \alpha_D > \alpha_E$

(C) $\alpha_A = \alpha_F > \alpha_C = \alpha_E > \alpha_B = \alpha_D$

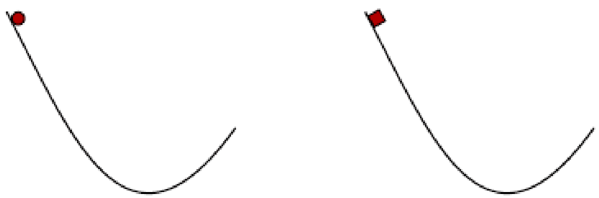
(D) $\alpha_D = \alpha_F > \alpha_A = \alpha_C > \alpha_B > \alpha_E$

10. Gina races her bike across a horizontal path. Suddenly, a squirrel runs in front of her. Gina slams on both her front and rear brakes, which results in the bike flipping over the front wheel and Gina flying over the handle bars. Which of the following do NOT contribute to an explanation of why the bike flips and Gina flies over the handlebars?



(A) Gina has a tendency to continue moving at a constant velocity, so while the bike stops, Gina continues her previous motion.
(B) Conservation of angular momentum of the bike and wheels indicates that if the wheels stop spinning in one direction, the bike must spin in the opposite direction.
(C) The large negative acceleration of the bike/rider system reduces the moment of inertia of the system, increasing the system's angular acceleration and causing a rotation of the bike.
(D) The force of the applied brakes at a distance from the center of mass of the bike and rider produces a net torque on the bike, causing a rotation bringing the back wheel of the back up.

12. A ball and a block of equal mass are situated on ramps with the same shape. The objects are released from the same height. The ball rolls without slipping, and the block travels without friction. After leaving the ramp, which object travels higher and why?



(A) The ball travels higher because it leaves the ramp with a higher speed.
(B) The ball travels higher because it gains rotational kinetic energy alongs its path.
(C) The block travels higher because it experiences no rotation.
(D) The block travels higher because energy is not conserved in a rolling system.