THE GRAPHICAL ANALYSIS OF MOTION

- **Kinematics** is the study of motion.
  - The term comes from the Greek root word *kinema*, from which we get the word "cinema," or moving pictures.
  - Kinematics is the study of how far (distance and displacement), how fast (speed and velocity), and the rate at which the "how fast" changes (acceleration).
- We say that an object moving in a straight line is moving in one dimension, and an object that is moving in a curved path (like a projectile) is moving in two dimensions.
  - If an object is moving with constant acceleration, then we relate all these quantities with a set of equations called the **kinematic equations**.
  - It is important that you are able to relate these equations to multiple representations like graphs, diagrams, and verbal descriptions of the motion.

Some helpful information:

- "Graph *a* versus *b*" means to graph the first quantity (*a*) on the *y* or vertical axis and to graph the second quantity (*b*) on the *x* or horizontal axis.
- The slope of a straight line graph is found from $\frac{\Delta y}{\Delta x}$, not $\frac{y}{x}$.
- The slope has units and you can determine those units by looking at the units on the *y*-axis divided by the units on the *x*-axis. The slope of a straight line has physical significance.
- The "area under the curve" has units, and those units of an "area" can be something other than area units.

DESCRIBING MOTION

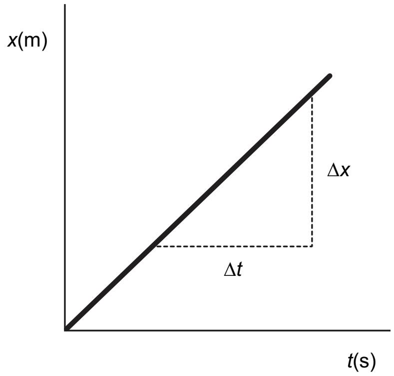
- An important concept is that all motion is relative. When we say that something has a given velocity, that velocity is relative to something else (these are called **reference frames**).
  - A car traveling to the east at 55 mph is doing so relative to Earth. Sitting in your bedroom you are not moving, so you are at rest but this is true only with respect to your room. Your bedroom and everything in it is rotating around the center of Earth.
  - Not only that, Earth itself is moving around the Sun in its orbit. The solar system is moving around the center of the galaxy, and the galaxy (and everything in it) is also moving away from the center of the universe!
  - If you are a passenger in an aircraft traveling at 300 mph over the surface of Earth, you are moving at 300 mph relative to Earth but have no motion relative to the plane unless you get up and start walking in the aisle. Then you might have a motion relative to the plane of, say, 2 mph.
  - Depending on whether you walk toward the front or the back of the aircraft, your motion relative to Earth could be 302 mph or 298 mph.
- Both analyses of motion, qualitative and quantitative, require the establishment of a frame of reference.
  - For the most part, we will assume a frame of reference with respect to Earth.
  - The direction of motion is determined by using a Cartesian coordinate system where the initial position is denoted as $x_o = 0$. If the object moves to the right, its direction is positive; if it moves to the left, its direction is negative.
- To develop conceptual understanding, you will need to translate from the graph to the actual motion, and from the actual motion to a graphical representation.

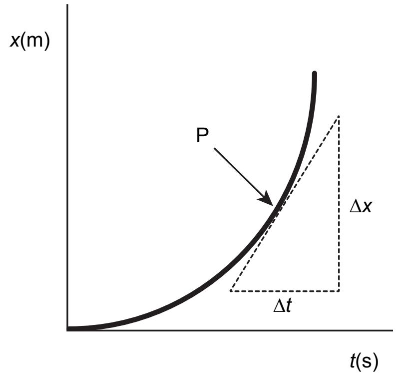
DISCUSSION OF IMPORTANT IDEAS		
Distance and Displacement	*x* or *y*	Distance *x* can be defined as total length moved. If you run around a circular track, you have covered a distance equal to the circumference of the track. Distance is a *scalar*, which means it has no direction associated with it.
	Δ*x* or Δ*y*	Displacement, Δ*x*, is a vector. A *vector* has both magnitude and direction. *Displacement* is defined as the straight-line distance between two points, and it is a vector that points from an object's initial position x_0 toward its final position x_f. In our previous example, if you run around a circular track and end up at the same place you started your displacement is zero because there is no distance between your starting point and your ending point. Displacement is often written in its scalar form as simply *x*.
Speed and Velocity	*v*	Speed and velocity are two closely related words. You might think that they are the same thing, but in physics we find that they are very different. *Speed* is a measure of how fast something moves. It is a rate: *rates* are quantities divided by time. In addition, speed is a scalar quantity.
		Velocity is also a rate, the rate that displacement changes with time. The bottom line here is that velocity is a vector. It has magnitude, just as speed does, but it also has a direction. When we talk about speed, we are not interested in the direction of motion. The speedometer in a car just tells us our speed, for example, 55 miles per hour. It does not indicate the direction of south, north, east, or west.
	$v = \frac{x}{t}$	*Average speed* is defined as the amount of distance a moving object covers divided by the amount of time it takes to cover that distance, $$v = \text{average speed} = \frac{\text{distance}}{\text{elapsed time}} = \frac{x}{t}$$ where *v* is speed, *x* is distance, and *t* is time.
	$v = \frac{\Delta x}{\Delta t}$	*Average velocity* is defined a little differently than average speed. Although average speed is the total change in distance divided by the total change in time, average velocity is the displacement divided by the change in time. Because velocity is a vector, we must define it in terms of another vector, displacement. Speed is the magnitude of velocity, that is, speed is a scalar and velocity is a vector. For example, if you are driving west at 55 miles per hour, we say that your speed is 55 mph and your velocity is 55 mph west. We will use the letter *v* for both speed and velocity in our calculations, and will take the direction of velocity into account when necessary, $$v = \frac{\Delta x}{\Delta t}$$ where Δx means the change of *x* or displacement and Δt means the elapsed time or time interval.

DISCUSSION OF IMPORTANT IDEAS		
Acceleration	$a = \frac{\Delta v}{\Delta t}$	*Acceleration* is a rate of a rate of change. It tells us how fast velocity is changing. For example, if you start from rest on the goal line of a football field and begin walking up to a speed of 1 m/s for the first second, then up to 2 m/s for the second second, then up to 3 m/s for the third second, you are speeding up with an average acceleration of 1 m/s for each second you are walking. In other words, you are changing your speed by 1 m/s for each second that you walk. $a = \frac{\Delta v}{\Delta t} = \frac{1\ \text{m/s}}{1\ \text{s}} = 1\ \text{m/s}^2$ If you start with a large velocity and slow down, you are still accelerating but your acceleration would be considered negative compared to the positive acceleration discussed previously.
	v_f and v_0	Usually, the change in speed $\pm\Delta v$ is calculated by taking the final speed v_f and subtracting the initial speed v_0. The initial and final speeds are called *instantaneous speeds* because they each occur at a particular instant in time and are not average speeds.
	g	An object is in *free fall* if it is falling freely under the influence of gravity. Any object, regardless of its mass, falls near the surface of Earth with an acceleration of 9.8 m/s^2, which we will denote with the letter g. This free fall acceleration assumes that there is no air resistance to impede the motion of the falling object, and this is a safe assumption unless you are told differently for a particular question on the exam. Because free fall acceleration is constant, we may use the kinematic equations to solve problems involving free fall. We simply need to replace the acceleration a with the specific free fall acceleration g in each equation. Remember that any time velocity and acceleration are in opposite directions (as when a ball is rising after being thrown upward), you must give one of them a negative sign. Notice that as the ball is rising the sign of the velocity and the acceleration is opposite because the ball is slowing down. On the other hand, the signs of the velocity and the acceleration are the same when the ball is falling because the ball is speeding up.

CONSTANT VELOCITY

Consider the position vs. time graphs shown:

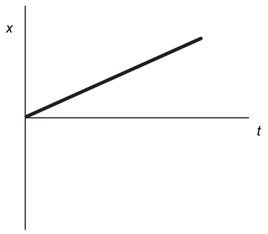


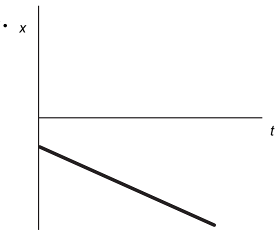


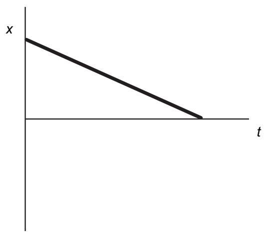
- The slope of the graph on the left is $\frac{\Delta x}{\Delta t}$, and is therefore velocity.
- The curved graph on the right indicates that the slope is changing. The slope of the curved graph is still velocity even though the velocity is changing, indicating the object is accelerating.
- The **instantaneous velocity** at any point on the graph (such as point P) can be found by drawing a tangent line at the point and finding the slope of the tangent line.
- The slope of a position-time graph is the same as the value (height) of velocity-time graph. Consequently, the value of a velocity-time graph is the same as the slope of a position-time graph.

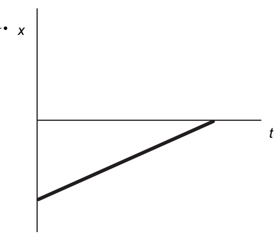
EXERCISE 1. POSITION VS. TIME GRAPHS

Qualitatively describe the motion depicted in the following position vs. time graphs:

a.


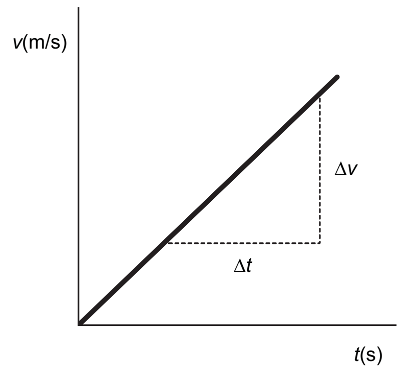
c.


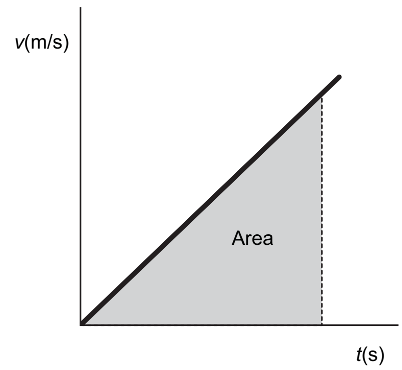
b.


d.


CONSTANT VELOCITY

Consider the velocity vs. time graphs shown:



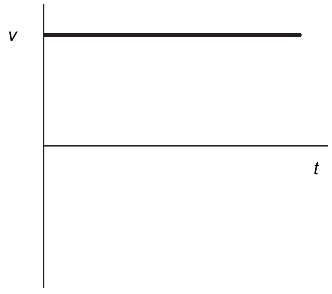


- The slope of the velocity vs. time graph on the left is $\frac{\Delta v}{\Delta t}$, and is therefore acceleration.
- The area under the velocity vs. time graph on the right has units of $\frac{\text{m}}{\text{s}} \times \text{s} = \text{m}$, and is therefore displacement.
- The slope of a velocity-time graph is the same as the value of an acceleration graph. Consequently, the value of an acceleration-time graph is the same as the slope of a velocity-time graph.

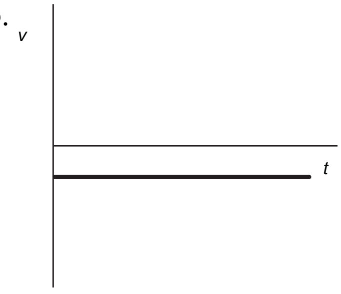
EXERCISE 2. VELOCITY VS. TIME GRAPHS

Qualitatively describe the motion depicted in the following velocity vs. time graphs:

a.



b.

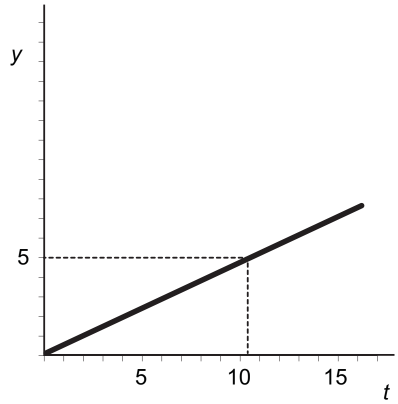


QUANTITATIVE APPROACH

- The next step is to calculate the slope of a position vs. time graph, and understand that the value obtained is the average velocity. When the velocity is constant, the average velocity over any time interval is equal to the instantaneous velocity at any time.
- The area under the curve of a velocity vs. time graph is the **displacement**.
  - When displacement is plotted on the *y*-axis and time is plotted on the *x*-axis, the curve is a straight line.
  - Happiness is a straight line because once you have a straight line, you now have an equation:

$$y = mx + b$$

where m is the slope and b is the y-intercept. The slope is the change in y divided by the change in x (otherwise known as "the rise over the run").



- Because we are graphing displacement on the *y*-axis, the change in *y* is simply the change in displacement, or Δy. We have Δt for the *x*-axis, so the slope is

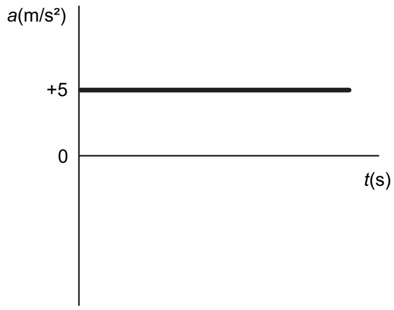
$$m = \frac{\Delta y}{\Delta x} = \frac{\Delta y}{\Delta t}$$

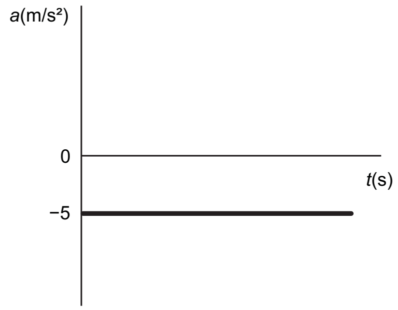
But $\frac{\Delta y}{\Delta t}$ is the velocity v, therefore the slope of the position vs. time graph is the velocity.
  - Example: What is the velocity of the object whose motion is depicted in the previous graph?

$$m = \frac{\Delta y}{\Delta x} = \frac{5\text{ m} - 0}{10.4\text{ s} - 0} = 0.48\text{ m/s}$$

ACCELERATED MOTION

- Because the AP Physics 1 exam generally deals with constant acceleration, any graph of acceleration vs. time on the exam would likely be a straight horizontal line:





  - This graph on the left tells us that the acceleration of this object is positive.
  - If the object were accelerating negatively, the horizontal line would be below the time axis, as shown in the graph on the right.
- If an object is undergoing a constant acceleration, we can analyze the motion and come up with several equations that will describe the motion. Start with the equation for acceleration:

$$a = \frac{v - v_0}{t - t_0}$$

Let $t_i = 0$, $v_0 = 0$:

$$a = \frac{v - v_0}{t}$$

$$v = v_0 + at$$

- So here are the kinematic motion equations. These will be provided to you on the AP Physics 1 exam. The following table shows their form on the test equation sheet.

KINEMATIC MOTION EQUATIONS	
$v_x = v_{x0} + a_x t$	*v* as a function of time
$x = x_0 + v_{x_0} t + \frac{1}{2} a_x t^2$	*x* as a function of time
$v_x^2 = v_{x_0}^2 + 2a_x (x - x_0)$	*x* as a function of velocity

AP TYPE QUESTION 1

You are asked to experimentally determine the acceleration of a skier traveling down a snow-covered hill of uniform slope as accurately as possible. Which combination of equipment and equation would be most useful in your experimental design?

a. Tape measure, stopwatch, $x = x_0 + v_{x_0} t + \frac{1}{2} a_x t^2$

b. Photogates, stopwatch, $v_x^2 = v_{x_0}^2 + 2a_x(x - x_0)$

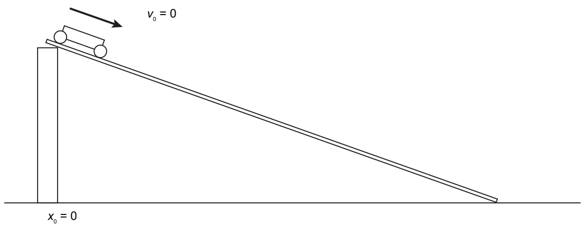
c. Radar gun, tape measure, $v_x = v_{x_0} + a_x t$

d. Photogates, radar gun, $v = \frac{v_{x_0} - v_x}{2}$

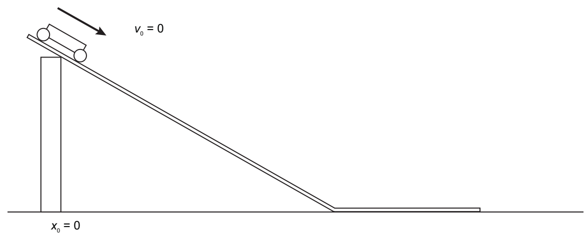
EXERCISE 3. ACCELERATED MOTION

A cart is rolling along a series of level and inclined tracks. Draw qualitative graphs of position vs. time, velocity vs. time, and acceleration vs. time.

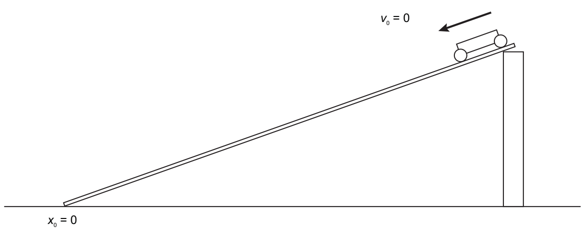
a.



d.

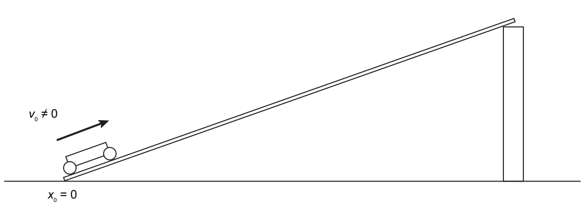


b.



e.

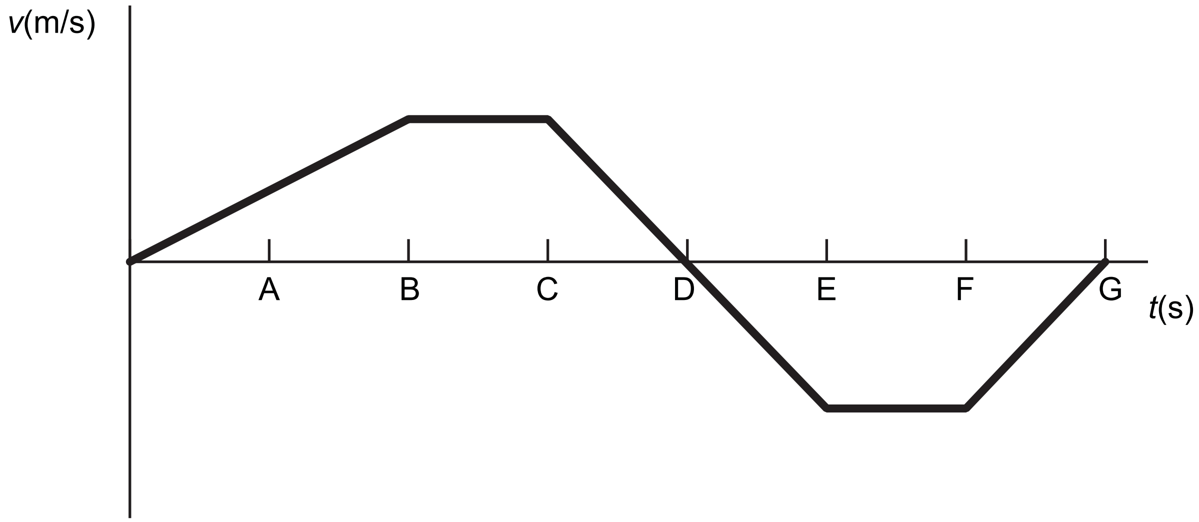
c.



INTERPRETING GRAPHS

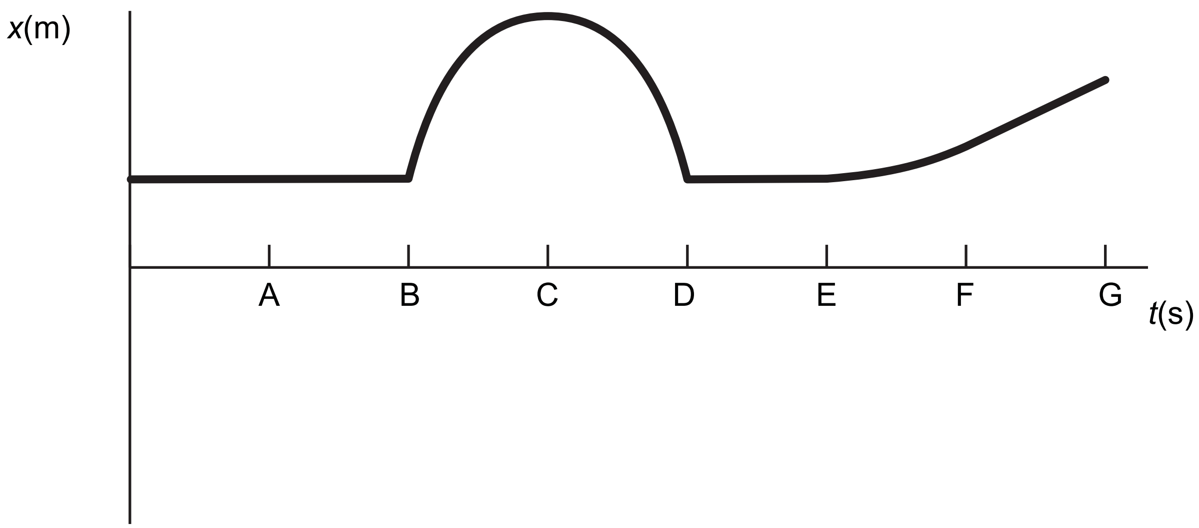
EXERCISE 4. INTERPRETING GRAPHS

Qualitatively describe the motion of an object at the different time intervals depicted in the following velocity vs. time graph:



EXERCISE 5. INTERPRETING GRAPHS

Qualitatively describe the motion of an object at the different time intervals depicted in the following position vs. time graph:

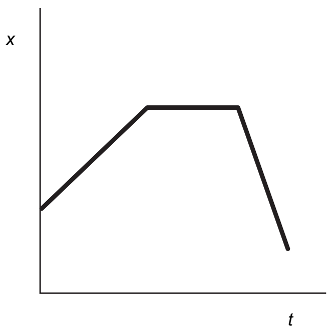


QUANTITATIVE APPROACH

- The quantitative approach includes the following calculations:
    - The slope of the tangent of a position vs. time graph defines instantaneous velocity.
    - The slope of the velocity vs. time graph is the average acceleration.
    - The area under the velocity vs. time graph is the displacement.
    - The area under the acceleration vs. time graph is the change in velocity.

EXERCISE 6. QUANTITATIVE APPROACH

In concise, coherent statements describe the motion depicted by Graph (a) and Graph (b). Sketch a velocity vs. time graph for Graph (a) and a position vs. time graph for Graph (b).

a.


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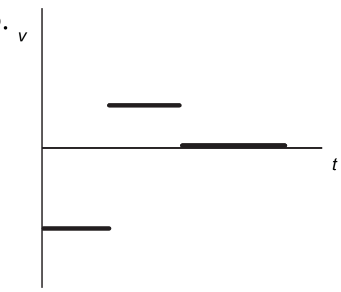
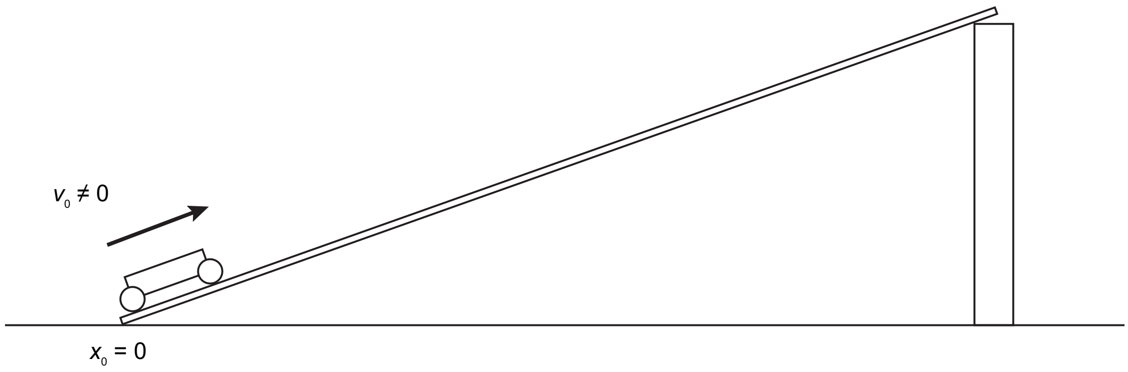


AP TYPE QUESTION 2

An object slides up and then down a frictionless track, as shown in the diagram. The object starts with a velocity greater than zero.



a. In a clear, concise paragraph describe the motion of the object.

b. Sketch the corresponding position vs. time graph, velocity vs. time graph, and the corresponding acceleration vs. time graph.

c. Qualitatively describe another type of motion that would produce the same shape on a position vs. time graph.

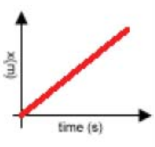
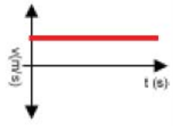
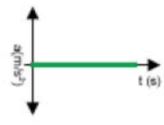
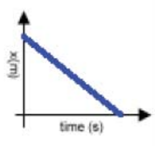
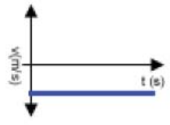
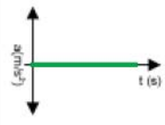
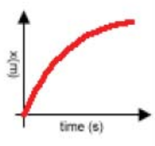
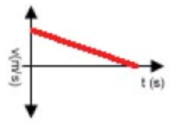
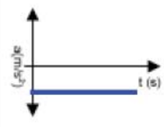
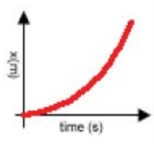
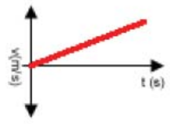
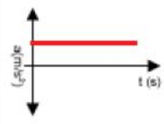
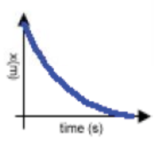
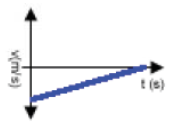
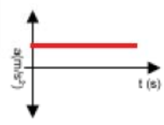
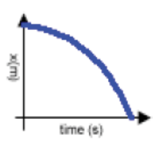
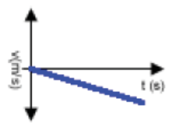
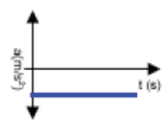
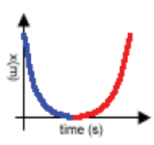
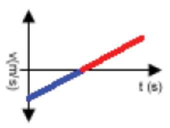
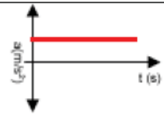
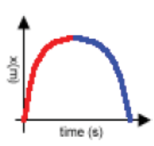
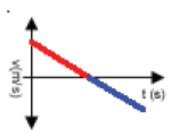
AP TYPE QUESTION 2 (CONTINUED)

d. When the object gets to the top of the ramp, a student measures the time it takes for the object to travel various displacements down the ramp using a stopwatch. Three consecutive trials are measured and the data is recorded as shown. Describe qualitatively and quantitatively how the student could determine the acceleration of the object.

Displacement (m)	Average Time (s)
0	0
0.2	0.68
0.4	0.98
0.6	1.18
0.8	1.38
1	1.52

SUMMARY OF MOTION INTERPRETATION

Graph of Motion Interpretation

Scenario *R.P. means Reference Point, i.e. the motion detector.*	Position vs. Time Graph *Draw the shape of the graph.*	Slope is.... Sign (+, – or zero)	Slope is.... Constant	Slope is.... Changing (Both): Increasing	Slope is.... Changing (Both): Decreasing	Velocity vs. Time Graph *Draw the shape of the graph*	Slope is.... Sign (+, – or zero)	Slope is.... Constant	Slope is.... Changing (Both): Increasing	Slope is.... Changing (Both): Decreasing	Acceleration vs. Time Graph *Draw the shape of the graph*	NET FORCE (+, - or zero)
Traveling AWAY from the R.P. at a CONSTANT RATE. (constant velocity)		*Positive*	*Constant*				*Positive*	*Constant*				
Traveling TOWARDS the R.P. at a CONSTANT RATE. (constant velocity)		*Negative*	*Constant*				*Negative*	*Constant*				
Traveling AWAY from the R.P. at a DECREASING RATE. (slowing down)		*Positive*			*Decreasing*		*Positive*	*Constant*				
Traveling AWAY from the R.P. at a INCREASING RATE. (speeding up)		*Positive*		*Increasing*			*Positive*	*Constant*				
Traveling TOWARDS the R.P. at a DECREASING RATE. (slowing down)		*Negative*			*Decreasing*		*Negative*	*Constant*				
Traveling TOWARDS the R.P. at a INCREASING RATE (speeding up)		*Negative*		*Increasing*			*Negative*	*Constant*				
Traveling TOWARDS then AWAY from the R.P. at a CHANGING RATE		*Neg. → Pos.*		*Increasing*	*Decreasing then*		*Constant*					
Traveling AWAY then TOWARDS the R.P. at a CHANGING RATE		*Pos. → Neg.*		*Increasing*	*Decreasing then*		*Constant*