

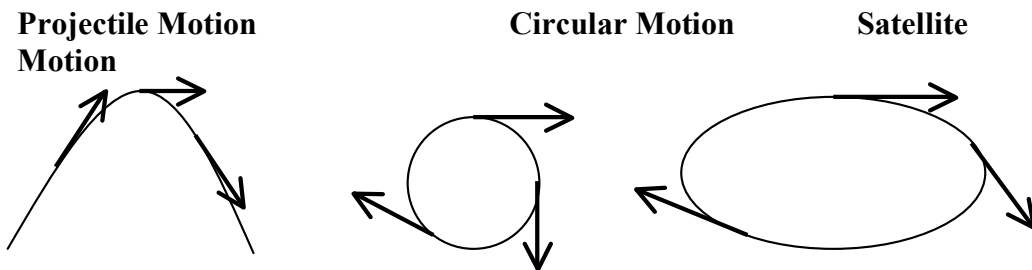
# AP Physics – Circular Motion and Gravity

**Frequency:** How often a repeating event happens. Measured in revolutions per second. Time is in the denominator.

**Period:** The time for one revolution.

**Speed:** Velocity is constantly changing when object travels along circular path. It can have a constant speed.

**Velocity:** However, you can measure instantaneous velocity for a point on the curve. Instantaneous velocity in any type of curved motion is tangent to the curve.  
**Tangential Velocity.**



The equation for speed and tangential velocity is the same  $v = \frac{2\pi r}{T}$

$T$  is the period, the time for one revolution, and  $r$  is the radius of the circular path.

**Acceleration:** *Centripetal Acceleration.* Due to inertia objects would follow the tangential velocity. But, they don't. The direction is being changed toward the center of the circle, or to the foci. In other words they are being accelerated toward the center.  $a_c = \frac{v^2}{r}$  Centripetal means **center seeking**.

**Force:** If an object is changing direction (accelerating) a force must be acting on it. The first law says that objects will have a constant velocity unless acted upon by an outside force. The force that changes the direction, therefore causing the circular motion is called the **centripetal force**.  $F_c = ma_c$

$$F_c = m \frac{v^2}{r}$$

1. As always, ask what the object is doing. **Changing direction, accelerating, toward the center, force centripetal.**
2. Set the direction of motion as positive. **Toward the center is positive, since this is the desired outcome.**
3. Identify the sum of force equation. In circular motion  $F_c$  is the sum of force.  $F_c$  can be any of the previous forces. If gravity is causing circular motion then  $F_c = F_g$ . If friction, then  $F_c = F_{fr}$ .

4. Substitute the relevant force equations and solve. For  $F_c$  substitute  $m\frac{v^2}{r}$

**The centripetal force equation is not provided on the AP test, so be prepared to derive it. You will be given the centripetal acceleration equation, so just use  $F = ma$  and plug in the centripetal acceleration.**

**Gravity:** Gravity is a property of matter that causes two objects to attract each other. The force is significant only when one of the objects has a very large mass. Sir Isaac Newton defined gravity in his **law of universal gravitation**.

$$F_g = G \frac{m_1 m_2}{r^2} \quad G \text{ is the universal gravitational constant.}$$

$$G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$$

**Acceleration of gravity:** To find the acceleration of gravity, combine the universal law of gravity and the second law:

$$F_g = G \frac{m_1 m_2}{r^2} \quad \text{and} \quad F_g = mg \quad mg = G \frac{m_1 m_2}{r^2} \quad g = G \frac{m}{r^2}$$

$r$  is not a radius, it is the distance between the objects measured from center to center. Is the problem asking for the height of a satellite above earth's surface? After you get  $r$  from the equation subtract earth's radius. Are you given height above the surface? Add the earth's radius to get  $r$  and then plug this in. Think center to center.

**Inverse Square Law:** The force of gravity is inversely proportional to the square of the distance between the two objects. If  $r$  doubles ( $\times 2$ ), invert to get  $\frac{1}{2}$  and then square it to get  $\frac{1}{4}$ . Thus gravity is  $\frac{1}{4}$  its original value. If the distance is cut in half, the force would increase by the square of the distance, or 4 times.

**Apparent Weight:** This is a consequence of your inertia. When an elevator, jet airplane, rocket, etc. accelerates upward the passenger wants to stay put due to inertia and is pulled down by gravity. The elevator pushes up and you feel heavier. Add the acceleration of the elevator to the acceleration of gravity  $F_{g \text{ apparent}} = mg + ma$ . If the elevator is going down subtract  $F_{g \text{ apparent}} = mg - ma$ . If the elevator is falling you will feel weightless  $g = a$  so  $F_{g \text{ apparent}} = 0$ . This same phenomenon works in circular motion. Your inertia wants to send you flying at the tangential velocity. You feel pressed up against the side of the car on the outside of the turn. So you think there is a force directed outward. This false non-existent force is really your inertia trying to send you out of the circle. The side of the car keeps you in moving in a circle just as the floor of the elevator moves you up. The car is forced to the center of the turn. No force exists to the outside. However, it feels like gravity, just like your inertia in the accelerating elevator makes you feel heavier. You are feeling  $g$ 's similar to what fighter pilots feel when turning hard. It is not your real weight, but rather what you appear to weight, *apparent weight*.

**Object in Orbit:** Satellite, moon, planet, etc. moves around object in circular path. Requires centripetal force,  $F_C$ , which is provided by Gravity.

You need to be prepared to derive the formulas for the period and orbital velocity for satellites. Do this by using the centripetal force equation and the law of universal gravitation.

$$F = G \frac{m_1 m_2}{r^2} \quad \text{also} \quad F_C = \frac{m_2 v^2}{r} \quad \text{Put Together:} \quad G \frac{m_1 m_2}{r^2} = \frac{m_2 v^2}{r}$$

$$v^2 = \frac{G m_1}{r} \quad v = \sqrt{\frac{G m_1}{r}} \quad \text{Orbital velocity equation}$$

Mass of satellite cancels out.

Period of satellite:  $T = \text{period} \equiv \text{time for one orbit.}$

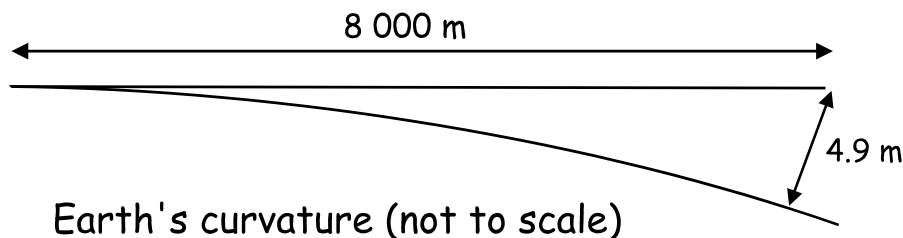
$$v = \frac{d}{t} \quad t = \frac{d}{v} \quad d \text{ is circumference of orbit} \quad d = 2\pi r$$

$$\text{So } t = \frac{2\pi r}{v} \quad \text{but } v \text{ is: } v = \sqrt{\frac{G m_1}{r}} \quad \text{Put together} \quad t = \frac{2\pi r}{\sqrt{\frac{G m_1}{r}}}$$

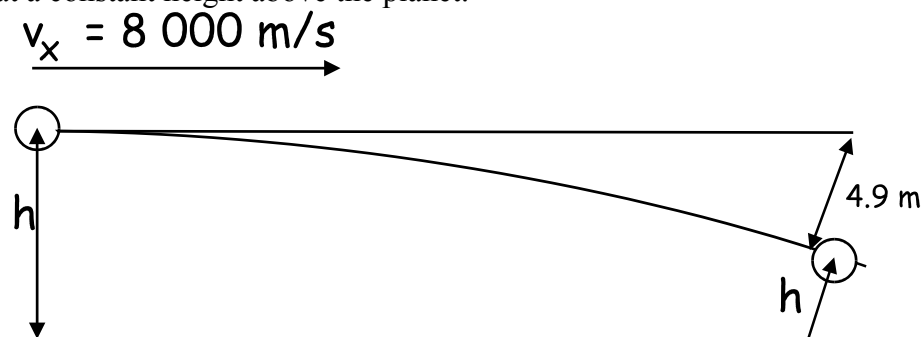
$$\text{Square both sides: } t^2 = \frac{4\pi^2 r^2}{\frac{G m_1}{r}} \quad \text{Clean up} \quad t^2 = 4\pi^2 r^2 \frac{r}{G m_1}$$

$$\text{Finally get: } t = 2\pi \sqrt{\frac{r^3}{G m_1}} \quad \text{Equation for period of a satellite.}$$

**How orbit works:** Earth's surface drops 4.9 m in 8 000 m. This is due to the curvature of the earth.



Satellite falls toward earth, but the surface of the earth curves away from the satellite at the same rate that the satellite is falling to earth. Result: satellite doesn't reach earth, but stays at a constant height above the planet.



**Free Fall:** A satellite is constantly falling towards the earth, so it is in a state of free fall. The Space Shuttle in orbit is in free fall. It and everything in it are also in free fall. This is why they appear to be weightless. The astronauts are weightless because they are falling towards the earth at the same rate that the space shuttle is falling.

### Equations on test

$$a_c = \frac{v^2}{r}$$

$$\sum F = F_{net} = ma$$

$$F_G = -\frac{Gm_1m_2}{r^2}$$

$a$  = acceleration

$F$  = Force

$G$  = universal gravitational constant

$m$  = mass

$r$  = distance

$t$  = time

$v$  = velocity or speed