**Simple Harmonic Motion**

Simple harmonic motion (SHM) is caused by a restoring force of some kind. The two most common restoring forces are springs and pendulums. For any SHM, there are 3 important points.

1. -A: A negative amplitude, or maximum displacement from the equilibrium position. Displacement is a maximum, velocity is zero, acceleration is a maximum (and in the opposite direction of displacement--displacement is away from the equilibrium position and force is toward it), potential energy is a maximum, kinetic energy is zero.
2. Equilibrium: at this point, the center of the motion, the net force acting on the object is zero. The displacement and acceleration are zero, the velocity is a maximum, the potential energy is a minimum, and the kinetic energy is a maximum. Once complete oscillation will move an object through the equilibrium position twice. A complete oscillation involves returning to the starting position and moving the same way at start.
3. +A : the same a -A, with the directions reversed. This is the maximum displacement in the other direction.

For a spring, the restoring force is *-kx*, the spring force. When measured from equilibrium, a vertical spring and a horizontal spring behave exactly the same. Gravity is not important for the motion of a mass on a spring, only the spring constant and the mass of the object. An oscillating spring will work in a "weightless" environment.

For a pendulum, the restoring force is the part of gravity that points back to equilibrium (the lowest point in its motion). The period of a pendulum depends only on the length of the pendulum and the acceleration due to gravity. The mass of the pendulum bob is unimportant for its motion.

**Multiple Choice**



1. A mass on the end of a spring oscillates with the displacement vs. time graph shown. Which of the following statements about its motion is INCORRECT?
2. The amplitude of the oscillation is 0.08 m.
3. The frequency of oscillation is 0.5 Hz.
4. The mass achieves a maximum in speed at 1 sec.
5. The period of oscillation is 2 sec.
6. The mass experiences a maximum acceleration at t=1.5 sec
7. The length of a simple pendulum with a period on Earth of one second is most nearly
8. 0.12 m
9. 0.25 m
10. 0.50 m
11. 1.0 m
12. 10.0 m
13. When an object oscillating in simple harmonic motion is at its maximum displacement from the equilibrium position. Which of the following is true of the values of its speed and the magnitude of the restoring force?

 Speed Restoring Force

1. Zero Maximum
2. Zero Zero
3. ½ maximum ½ maximum
4. Maximum ½ maximum
5. Maximum Zero
6. An object is attached to a spring and oscillates with amplitude A and period T, as represented on the graph. The nature of the velocity v and acceleration a of the object at time T/4 is best represented by which of the following?
7. v > 0, a > 0
8. v > 0, a < 0
9. v > 0, a = 0
10. v = 0, a < 0
11. v = 0, a = 0

Questions 5-6



A block oscillates without friction on the end of a spring as shown above. The minimum and maximum lengths of the spring as it oscillates are, respectively, xmin and xmax. The graphs below can represent quantities associated with the oscillation as functions of the length x of the spring.



1. Which graph can represent the total mechanical energy of the block-spring system as a function of x?
2. A
3. B
4. C
5. D
6. E
7. Which graph can represent the kinetic energy of the block as a function of x?
8. A
9. B
10. C
11. D
12. E



Questions 7-8

A sphere of mass m1, which is attached to a spring, is displaced downward from its equilibrium position as shown above left and released from rest. A sphere of mass m2, which is suspended from a string of length L, is displaced to the right as shown above right and released from rest so that it swings as a simple pendulum with small amplitude. Assume that both spheres undergo simple harmonic motion.

1. Which of the following is true for both spheres?
2. The maximum kinetic energy is attained as the sphere passes through its equilibrium position
3. The maximum kinetic energy is attained as the sphere reaches its point of release.
4. The minimum gravitational potential energy is attained as the sphere passes through its equilibrium position.
5. The maximum gravitational potential energy is attained when the sphere reaches its point of release.
6. The maximum total energy is attained only as the sphere passes through its equilibrium position.
7. If both spheres have the same period of oscillation, which of the following is an expression for the spring constant
8. L / m1g
9. g/ m2L
10. m1L/g
11. m2g/L
12. m1g/L



Questions 9-10

 A block on a horizontal frictionless plane is attached to a spring, as shown above. The block oscillates along the x-axis with simple harmonic motion of amplitude A.

1. Which of the following statements about the block is correct?
2. At x = 0, its velocity is zero.
3. At x = 0, its acceleration is at a maximum.
4. At x = A, its displacement is at a maximum.
5. At x = A, its velocity is at a maximum.
6. At x = A, its acceleration is zero.
7. Which of the following statements about energy is correct?
8. The potential energy of the spring is at a minimum at x = 0.
9. The potential energy of the spring is at a minimum at x = A.
10. The kinetic energy of the block is at a minimum at x =0.
11. The kinetic energy of the block is at a maximum at x = A.
12. The kinetic energy of the block is always equal to the potential energy of the spring.

**Free Response:**

1. The simple pendulum above consists of a bob hanging from a light string. You wish to experimentally determine the frequency of the swinging pendulum.
2. By checking the line next to each appropriate item on the list below, select the equipment that you would need to do the experiment. Meterstick Protractor Additional string Stopwatch Photogate Additional masses
3. Describe the experimental procedure that you would use. In your description, state the measurements you would make, how you would use the equipment to make them, and how you would determine the frequency from those measurements.
4. You next wish to discover which parameters of a pendulum affect its frequency. State one parameter that could be varied, describe how you would conduct the experiment, and indicate how you would analyze the data to show whether there is a dependence.
5. After swinging for a long time, the pendulum eventually comes to rest. In terms of energy considerations, explain why the pendulum came to rest.
6. The spring mass system shown in the diagram is placed in simple harmonic motion by pulling the mass down 25.0 cm and releasing it. A motion detector is placed directly below the mass. The mass is 200.0 g and the spring has a constant of 100.0 N/m. Assume g = 10.0 m/s2.



1. The position versus time graph for the mass is shown here as it spans two complete oscillations.



1. At what times is the mass moving upward at maximum speed? Give your answer in terms of the period, T. Justify your answer
2. At what times is the mass instantaneously at rest? Give your answer in terms of the period, T. Justify your answer.
3. On the axes provided, sketch the velocity, v, as a function of time and the acceleration, a, as a function of time for the mass as it completes two oscillations. The length of each time axis represents two periods.



1. Simple harmonic motion occurs when there is a restoring force that is proportional to the displacement of the mass from equilibrium. Students wish to compare their graph of position versus time and a function that describes the motion of the mass. Consider the function

x = *A*cos 2( π *ft*)

Where $f=^{1}/\_{T}$ so that

$$x=A\cos(\left(\frac{2πt}{T}\right))$$

1. What are the units for the term $\left(\frac{2πt}{T}\right)$?
2. Show quantitatively whether or not the function matches the graph for the values of x at *t =0 s* and t = T/2 s?
3. While the spring/mass system is in motion, calculate the total mechanical energy of the system.